A Study on Integral Transform of the Generalized Lommel Wright K-Function

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Abstract – *In the present work we introduces an integral transform of the lommel wright k- function. These transform are expressed in terms of the wright hypergeometric k-function. various interesting transform as the consequence of this method are obtained. which plays an important role to solve the differential equation and also some relations and results related to this generalized lommel wright k-function.*

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1.INTRODUCTION

In recent years the fractional calculus has become one of the most rapidly growing research subject of mathematical analysis due to its numerous applications in various parts of science as well as mathematics.

The transform defined by the following Integral equation

$$\mathrm{B}[\mathrm{f}(\mathrm{x});\mathrm{a},\mathrm{b}] = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1} f(x)\,dx \qquad (1)$$

is called the Euler Beta transform with p as a complex parameter.

The Laplace transform of a function f(x) is defined by

$$\mathrm{L}\{\mathrm{f}(\mathrm{p})\} = \int_0^\infty e^{-xp} f(x)\,dx \qquad (2)$$

Where p is a complex parameter.

The transform

$$R_v f(x); p = g(p,v) = \int_0^\infty (px)^{1/2} K_v(px) f(x)\,dx \qquad (3)$$

is called the K-transform with p as a complex parameter and $K_v(px)$ is called the modified Bessel function of the third kind or the Macdonald function,see(Mathai et al,2010,p.53).

The Hankel transform of a function f(x) denoted by g(p,v) defined as

$$g(p,v) = \int_0^\infty (px)^{1/2} J_v(px) f(x)\,dx\,, p > 0 \qquad (4)$$

where $J_v(px)$ is called the Bessel-Maitland function or the maitland Bessel function (Mathai et al, 2010, p.22 and p.56).

Next for the result of this paper. we take The Wright Hypergeometric function in series form [9], denoted by $({}_p\psi_q(z))$ is define as:

$${}_p\psi_q\left[\begin{matrix}(\alpha_1,A_1)\cdots(\alpha_p,A_p) \\ (\beta_1,B_1)\cdots(\beta_q,B_q)\end{matrix} \;;\quad z\right] = \sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}\Gamma(\alpha_j+A_j n)z^n}{\prod_{j=1}^{q}\Gamma(\beta_j+B_j n)n!}$$

where the cofficient $(A_1 \ldots\ldots A_p)$ and $(B_1 \ldots\ldots B_q)$ are positive real numbers such that.

$$1+\sum_{j=1}^{q} B_j \; - \sum_{j=1}^{p} A_j \geq 0$$

Gehlot and prajapati defined the lommel wright k function as follows[5].

For $\left(k \in R^{+}, w, a_i, b_j \in C, \& A_i, B_j \in R(A_i, B_j)\&\right)$ *where* $(i = 1,2, \dots, \theta)$ *and* $\left((a_i + A_i n), (a_i + B_j n) \in C\right)$.

We take the Wright Hypergeometric k-function in series form[3],denoted by $\,_p\psi_q^k$ is define as:

$$\,_p\psi_q^k\left[\begin{matrix}(\alpha_1, kA_1)\cdots(\alpha_p, kA_p) \\ (\beta_1, kB_1)\cdots(\beta_q, kB_q)\end{matrix} \quad ; \quad z\right] = \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{p} \Gamma_k(\alpha_j + A_j nk)}{\prod_{j=1}^{q} \Gamma_k(\beta_j + B_j nk)} \frac{z^n}{n!} \qquad (5)$$

Where the cofficients $(A_1 \dots . A_p)$ and $(B_1 \dots . B_q)$ are positive real number such that

$$1+\sum_{j=1}^{q} \frac{B_i}{k} - \sum_{j=1}^{p} \frac{A_j}{k} \geq 0$$

and slightly generalized form is

$$\,_p\psi_q^k\left[\begin{matrix}(\alpha_1, kA_1)\cdots(\alpha_p, kA_p) \\ (\beta_1, kB_1)\cdots(\beta_q, kB_q)\end{matrix} \quad ; \quad z\right] = \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{p} \Gamma_k(\alpha_j)}{\prod_{j=1}^{q} \Gamma_k(\beta_j)} =_p F_q^k\left[\begin{matrix}(\alpha_1, \mathrm{k})\cdots(\alpha_p, \mathrm{k}) \\ (\beta_1, \mathrm{k})\cdots(\beta_q, \mathrm{k})\end{matrix} \quad ; \quad z\right] \qquad (6)$$

Where $\,_pF_q^k$ is the generalised hypergeometric k-function defined by

$$\,_pF_q^k\left[\begin{matrix}(\alpha_1, \mathrm{k})\cdots(\alpha_p, \mathrm{k}) \\ (\beta_1, \mathrm{k})\cdots(\beta_q, \mathrm{k})\end{matrix} \quad ; \quad z\right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_{n,k}\cdots(\alpha_p)_{n,k}}{(\beta)_{n,k}\cdots(\beta)_{n,k}} \frac{z^n}{n!} \qquad (7)$$

The series representation of the generalized lommel wright k-function is defined by

$$J_{\aleph,\hbar,k}^{\psi,m}(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma_k(\hbar + k + nk)^m \Gamma_k(\aleph + \hbar + n\psi + k)} \left(\frac{z}{2}\right)^{2n+\frac{\aleph+2\hbar}{k}} \qquad (8)$$

Where $(z \in C/(-\infty, 0], m \in N, \aleph, \hbar \in C, k \in N^{+}, \psi > 0)$ and $\Gamma_k(z)$ is the k-gamma function introduced by Diaz and Pariguan [6] given by

$$\Gamma_k(z) = \lim_{n\to\infty} \frac{(n)!\, k^n\, (nk)^{\frac{\omega}{k}-1}}{(w)_{n,k}} \qquad (9)$$

with k-pochhammer symbol $(z)_{n,k} = z(z+k)(z+2k) \dots .(z+(n-1)k)\; z \in C, k \in N^{+}$

The classical Euler gamma function and gamma k-function are related with following relation

$$\Gamma_k(z) = k^{\frac{z}{k}-1}\left(\Gamma\frac{z}{k}\right)$$

for k=1 generalized lommel wright k-function reduce in generalized lommel wright function. For m=1 reducing in generalized bessel maitland k-function. for m=k=1 reducing in bessel maitaland function. for m=v=k=1 and $\mu = 0$ reducing in claasical bessel function.

Theorem 2.1.(Euler Beta Transform)

Let $(z \in C/(-\infty,0], m \in N, \aleph, \hbar \in C, \psi > 0 \; and \; k \in N^+)$ then the Euler beta transform of the generalized lommel wright k-function is

$$\int_0^\infty t^{\alpha-1}(1-t)^{\beta-1} J_{\aleph,\hbar,k}^{\psi,m}(xt^\sigma)dt = k^\beta \Gamma\beta\left(\frac{x}{2}\right)^{\frac{\aleph+2\hbar}{k}}$$

$$\times \; {}_2\Psi_{m+2}^k\left[\begin{matrix}(k,k)\;(\alpha k+\sigma\aleph+2\sigma\hbar,2\sigma k)\\(\hbar+k,k)\cdots(\hbar+k,k)(\aleph+\hbar+k,\psi)(\alpha k+\beta k+\sigma\aleph+2\sigma\hbar,2\sigma k)\end{matrix}\;;\;\left(-\frac{x^2}{4k}\right)\right] \qquad (10)$$

Proof- On using(8) in the integrand of (10) , we get

$$\int_0^\infty t^{\alpha-1}(1-t)^{\beta-1} J_{\aleph,\hbar,k}^{\psi,m}(xt^\sigma)dt =$$

$$\times \sum_{n=0}^\infty \frac{(-1)^n}{\Gamma_k(\hbar+k+nk)^m\,\Gamma_k(\aleph+\hbar+n\psi+k)}\left(\frac{x}{2}\right)^{2n+\frac{\aleph+2\hbar}{k}} \int_0^\infty t^{\alpha+\sigma\left(2n+\frac{\aleph+2\hbar}{k}\right)-1}(1-t)^{\beta-1}\,dt \qquad (11)$$

Now using (1) in the above equation, we get

$$\int_0^\infty t^{\alpha-1}(1-t)^{\beta-1} J_{\aleph,\hbar,k}^{\psi,m}(xt^\sigma)dt =$$

$$k^\beta \Gamma\beta\left(\frac{x}{2}\right)^{\frac{\aleph+2\hbar}{k}} \sum_{n=0}^\infty \frac{\Gamma_k(nk+k)\,\Gamma_k(\alpha k+\sigma(2nk+\aleph+2\hbar))}{n!\,\Gamma_k(\hbar+k+nk)^m\,\Gamma_k(\aleph+\hbar+n\psi+k)\,\Gamma_k(\alpha k+\beta k+\sigma(2nk+\aleph+2\hbar))}\left(-\frac{x^2}{4k}\right)^n \qquad (12)$$

$$= k^\beta \Gamma\beta\left(\frac{x}{2}\right)^{\frac{\aleph+2\hbar}{k}} \; {}_2\Psi_{m+2}^k\left[\begin{matrix}(k,k)\;(\alpha k+\sigma\aleph+2\sigma\hbar,2\sigma k)\\(\hbar+k,k)\cdots(\hbar+k,k)(\aleph+\hbar+k,\psi)(\alpha k+\beta k+\sigma\aleph+2\sigma\hbar,2\sigma k)\end{matrix}\;;\;\left(-\frac{x^2}{4k}\right)\right] \qquad (13)$$

Theorem 2.2.(Laplace Transform)

Let $(z \in C/(-\infty,0], m \in N, \aleph, \hbar \in C, \psi > 0 \; and \; k \in N^+)$ then the laplace transform of the generalized lommel wright k-function is

$$\int_0^\infty t^{\alpha-1} e^{-\delta t} J_{\aleph,\hbar,k}^{\psi,m}(xt^\sigma)dt$$

$$= \frac{k^{1-\alpha}}{\delta^\alpha}\left(\frac{x}{2(\delta k)^\sigma}\right)^{\frac{\aleph+2\hbar}{k}} \; {}_2\Psi_{m+1}^k\left[\begin{matrix}(k,k)\;(\alpha k+\sigma\aleph+2\sigma\hbar,2\sigma k)\\(\hbar+k,k)\cdots(\hbar+k,k)(\aleph+\hbar+k,\psi)\end{matrix}\;;\;\left(-\frac{x^2}{4k(\delta k)^{2\sigma}}\right)\right] \qquad (14)$$

Proof: On using (8) in the integrand of (14),we get

$$\int_0^\infty t^{\alpha-1} e^{-\delta t} J_{\aleph,\hbar,k}^{\psi,m}(xt^\sigma)\,dt$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma_k(\hbar+k+nk)^m \Gamma_k(\aleph+\hbar+n\psi+k)} \left(\frac{x}{2}\right)^{2n+\frac{\aleph+2\hbar}{k}} \int_0^\infty e^{-\delta t} t^{\alpha+\sigma(2n+\frac{\aleph+2\hbar}{k})-1}\,dt \tag{15}$$

Now using (2) in the above equation we get

$$\int_0^\infty t^{\alpha-1} e^{-\delta t} J_{\aleph,\hbar,k}^{\psi,m}(sxt^\sigma)\,dt$$

$$= \frac{k^{1-\alpha}}{\delta^\alpha}\left(\frac{x}{2(\delta k)^\sigma}\right)^{\frac{\aleph+2\hbar}{k}} \sum_{n=0}^{\infty} \frac{\Gamma_k(nk+k)\,\Gamma_k(\alpha k+\sigma(2nk+\aleph+2\hbar)}{n!\,\Gamma_k(\hbar+k+nk)^m \Gamma_k(\aleph+\hbar+n\psi+k)} \left(-\frac{x^2}{4k(\delta k)^{2\sigma}}\right)^n \tag{16}$$

$$= \frac{k^{1-\alpha}}{\delta^\alpha}\left(\frac{x}{2(\delta k)^\sigma}\right)^{\frac{\aleph+2\hbar}{k}} {}_2\Psi_{m+1}^{k}\left[\begin{matrix} (k,k)\,(\alpha k+\sigma\aleph+2\sigma\hbar, 2\sigma k) \\ (\hbar+k,k)\cdots(\hbar+k,k)(\aleph+\hbar+k,\psi)\end{matrix} ;\ \left(-\frac{x^2}{4k(\delta k)^{2\sigma}}\right)\right] \tag{17}$$

Theorem 2.3. (K-Transform)

Let $(z \in C/(-\infty,0], m \in N, \aleph, \hbar \in C, \psi > 0 \text{ and } k \in N^+)$ then the k-transform of the generalized lommel wright k-function is

$$\int_0^\infty t^{\alpha-1} k_\eta(at) J_{\aleph,\hbar,k}^{\psi,m}(xt^\sigma)\,dt$$

$$= \frac{k^{1-\frac{\alpha+\eta}{2}}}{4}\left(\frac{2}{a}\right)^\alpha \left(\frac{x}{2}\left(\frac{2}{a\sqrt{k}}\right)^\sigma\right)^{\frac{\aleph+2\hbar}{k}} {}_2\Psi_{m+1}^{k}\left[\begin{matrix} (k,k)\,(\alpha k+\sigma\aleph+2\sigma\hbar, 2\sigma k) \\ (\hbar+k,k)\cdots(\hbar+k,k)(\aleph+\hbar+k,\psi)\end{matrix} ;\ \left(-\frac{x^2}{4k(\delta k)^{2\sigma}}\right)\right] \tag{18}$$

Proof: On using (8) in the integrand of (18), we get

$$\int_0^\infty t^{\alpha-1} k_\eta(at) J_{\aleph,\hbar,k}^{\psi,m}(xt^\sigma)\,dt$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma_k(\hbar+k+nk)^m \Gamma_k(\aleph+\hbar+n\psi+k)} \left(\frac{x}{2}\right)^{2n+\frac{\aleph+2\hbar}{k}} \int_0^\infty t^{\alpha+\sigma(2n+\frac{\aleph+2\hbar}{k})-1} k_\eta(at)\,dt \tag{19}$$

Now using (3) in the above equation we get

$$\int_0^\infty t^{\alpha-1} k_\eta(at) J_{\aleph,\hbar,k}^{\psi,m}(xt^\sigma)\,dt$$

$$= \frac{k^{1-\frac{\alpha+\eta}{2}}}{4}\left(\frac{2}{a}\right)^\alpha \left(\frac{x}{2}\left(\frac{2}{a\sqrt{k}}\right)^\sigma\right)^{\frac{\aleph+2\hbar}{k}} \sum_{n=0}^{\infty} \frac{\Gamma_k(nk+k)\,\Gamma_k(\alpha k+\sigma(2nk+\aleph+2\hbar)}{n!\,\Gamma_k(\hbar+k+nk)^m \Gamma_k(\aleph+\hbar+n\psi+k)} \left(-\frac{x^2}{4k(\delta k)^{2\sigma}}\right)^n \tag{20}$$

$$= \frac{k^{1-\frac{\alpha+\eta}{2}}}{4}\left(\frac{2}{a}\right)^\alpha \left(\frac{x}{2}\left(\frac{2}{a\sqrt{k}}\right)^\sigma\right)^{\frac{\aleph+2\hbar}{k}} {}_2\Psi_{m+1}^{k}\left[\begin{matrix} (k,k)\,(\alpha k+\sigma\aleph+2\sigma\hbar, 2\sigma k) \\ (\hbar+k,k)\cdots(\hbar+k,k)(\aleph+\hbar+k,\psi)\end{matrix} ;\ \left(-\frac{x^2}{4k(\delta k)^{2\sigma}}\right)\right] \tag{21}$$

Theorem 2.4.(Hankel Transform)

Let $(z \in C/(-\infty, 0], m \in N, \aleph, \hbar \in C, \psi > 0 \; and \; k \in N^+)$ then the hankel transform of the generalized lommel wright k-function is

$$\int_0^\infty t^{\alpha-1} J_\eta(at) J_{\aleph,\hbar,k}^{\psi,m}(xt^\sigma)dt = \frac{1}{2}\left(\frac{x}{2}\right)^{\frac{\aleph+2\hbar}{k}} \left(\frac{2}{ak}\right)^{\alpha+\sigma\left(\frac{\aleph+2\hbar}{k}\right)}$$

$$\times \; {}_2\Psi_{m+2}^k \left[\begin{matrix} (k,k) \left(\frac{\alpha k}{2} + \frac{\sigma \aleph}{2} + \frac{\eta k}{2} + \sigma\hbar, \sigma k\right) \\ (\hbar + k, k) \cdots (\hbar + k, k)(\aleph + \hbar + k, \psi) \left(k + \frac{\eta k}{2} - \frac{\alpha k}{2} - \frac{\sigma \aleph}{2} - \sigma\hbar, -\sigma k\right) \end{matrix} ; \left(-\frac{x^2}{4k}\left(\frac{4}{(ak)^2}\right)^\sigma\right) \right] \tag{22}$$

Proof: On using (8) in the integrand of (22), we get

$$\int_0^\infty t^{\alpha-1} J_\eta(at) J_{\aleph,\hbar,k}^{\psi,m}(xt^\sigma)dt$$

$$= \sum_{n=0}^\infty \frac{(-1)^n}{\Gamma_k(\hbar + k + nk)^m \Gamma_k(\aleph + \hbar + n\psi + k)} \left(\frac{x}{2}\right)^{2n+\frac{\aleph+2\hbar}{k}} \int_0^\infty t^{\alpha+\sigma\left(2n+\frac{\aleph+2\hbar}{k}\right)-1} J_\eta(at)\, dt \tag{23}$$

Now using (4) in the above equation we get

$$\int_0^\infty t^{\alpha-1} J_\eta(at) J_{\aleph,\hbar,k}^{\psi,m}(xt^\sigma)dt = \frac{1}{2}\left(\frac{x}{2}\right)^{\frac{\aleph+2\hbar}{k}} \left(\frac{2}{ak}\right)^{\alpha+\sigma\left(\frac{\aleph+2\hbar}{k}\right)}$$

$$\times \sum_{n=0}^\infty \frac{\Gamma_k(nk+k)\, \Gamma_k\left(\alpha k + \sigma(2nk + \aleph + 2\hbar) + \eta k/2\right.}{n!\, \Gamma_k(\hbar + k + nk)^m \Gamma_k(\aleph + \hbar + n\psi + k)\, \Gamma_k\left(2k + \eta k - \alpha k - \sigma(2nk + \aleph + 2\hbar)/2\right.} \left(-\frac{x^2}{4k}\left(\frac{2}{ak}\right)^{2\sigma}\right)^n \tag{24}$$

$$= \frac{1}{2}\left(\frac{x}{2}\right)^{\frac{\aleph+2\hbar}{k}} \left(\frac{2}{ak}\right)^{\alpha+\sigma\left(\frac{\aleph+2\hbar}{k}\right)}$$

$$\times \; {}_2\Psi_{m+2}^k \left[\begin{matrix} (k,k) \left(\frac{\alpha k}{2} + \frac{\sigma \aleph}{2} + \frac{\eta k}{2} + \sigma\hbar, \sigma k\right) \\ (\hbar + k, k) \cdots (\hbar + k, k)(\aleph + \hbar + k, \psi) \left(k + \frac{\eta k}{2} - \frac{\alpha k}{2} - \frac{\sigma \aleph}{2} - \sigma\hbar, -\sigma k\right) \end{matrix} ; \left(-\frac{x^2}{4k}\left(\frac{4}{(ak)^2}\right)^\sigma\right) \right] \tag{25}$$

Special Cases:

In this section, we get some integral formulas involving lommel wright k-function as follows.

Using m=1,the results (2.1) to (2.4) take of the form

Corollary 1.

Let $(z \in C/(-\infty, 0], \aleph, \hbar \in C, \psi > 0 \; and \; k \in N^+)$ Then by (10) the Euler Beta Transform, we obtain:

$$\int_0^\infty t^{\alpha-1}(1-t)^{\beta-1} J_{\aleph,\hbar,k}^{\psi}(xt^\sigma)dt$$

$$= k^\beta \Gamma\beta \left(\frac{x}{2}\right)^{\frac{\aleph+2\hbar}{k}} {}_2\Psi_3^k \left[\begin{matrix} (\alpha k + \sigma\aleph + 2\sigma\hbar, 2\sigma k) \; (k,k) \\ (\hbar + k, k) \; (\aleph + \hbar + k, \psi)(\alpha k + \beta k + \sigma\aleph + 2\sigma\hbar, 2\sigma k) \end{matrix} ; \left(-\frac{x^2}{4k}\right) \right] \tag{26}$$

Corollary 2.

Let$(z \in C/(-\infty,0], \aleph, \hbar \in C, \psi > 0 \; and \; k \in N^{+})$ Then by (14) the Laplace Transform, we obtain:

$$\int_0^\infty t^{\alpha-1} e^{-\delta t} J^{\psi}_{\aleph\hbar,k}(xt^{\sigma})\,dt$$

$$= \frac{k^{1-\alpha}}{\delta^{\alpha}} \left(\frac{x}{2(\delta k)^{\sigma}}\right)^{\frac{\aleph+2\hbar}{k}} {}_2\Psi_2^k \left[\begin{matrix} (\alpha k + \sigma\aleph + 2\sigma\hbar, 2\sigma k)\;(k,k) \\ (\hbar + k, k)\;(\aleph + \hbar + k, \psi) \end{matrix} \; ; \; \left(-\frac{x^2}{4k(\delta k)^{2\sigma}}\right) \right] \tag{27}$$

Corollary 3.

Let $(z \in C/(-\infty,0], \aleph, \hbar \in C, \psi > 0 \; and \; k \in N^{+})$ Then by (18) the K-Transform, we obtain:

$$\int_0^\infty t^{\alpha-1} k_{\eta}(at) J^{\psi}_{\aleph\hbar,k}(xt^{\sigma})\,dt$$

$$= \frac{k^{1-\frac{\alpha+\eta}{2}}}{4} \left(\frac{2}{a}\right)^{\alpha} \left(\frac{x}{2}\left(\frac{2}{a\sqrt{k}}\right)^{\sigma}\right)^{\frac{\aleph+2\hbar}{k}} {}_2\Psi_2^k \left[\begin{matrix} (\alpha k + \sigma\aleph + 2\sigma\hbar, 2\sigma k)\;(k,k) \\ (\hbar + k, k)\;(\aleph + \hbar + k, \psi) \end{matrix} \; ; \; \left(-\frac{x^2}{4k(\delta k)^{2\sigma}}\right) \right] \tag{28}$$

Corollary 4.

Let $(z \in C/(-\infty,0], \aleph, \hbar \in C, \psi > 0 \; and \; k \in N^{+})$ Then by (22) the Hankel Transform, we obtain:

$$\int_0^\infty t^{\alpha-1} J_{\eta}(at) J^{\psi}_{\aleph\hbar,k}(xt^{\sigma})\,dt$$

$$= \frac{1}{2}\left(\frac{x}{2}\right)^{\frac{\aleph+2\hbar}{k}} \left(\frac{2}{ak}\right)^{\alpha+\sigma\left(\frac{\aleph+2\hbar}{k}\right)} {}_2\Psi_3^k \left[\begin{matrix} \left(\frac{\alpha k}{2} + \frac{\sigma\aleph}{2} + \frac{\eta k}{2} + \sigma\hbar, \sigma k\right)\;(k,k) \\ (\hbar + k, k)\;(\aleph + \hbar + k, \psi)\left(k + \frac{\eta k}{2} - \frac{\alpha k}{2} - \frac{\sigma\aleph}{2} - \sigma\hbar, -\sigma k\right) \end{matrix} \; ; \; \left(-\frac{x^2}{4k}\left(\frac{4}{(ak)^2}\right)^{\sigma}\right) \right] \tag{29}$$

Letting k = 1 , we have the generalized Lommel-Wright function and the corresponding formulas are presented in subsequent corollaries.

Corollary 5.

Let $(z \in C/(-\infty,0], m \in N, \aleph, \hbar \in C \; and \; \psi > 0)$ Then by (10) the Euler Beta Transform, we obtain:

$$\int_0^\infty t^{\alpha-1}(1-t)^{\beta-1} J^{\psi,m}_{\aleph\hbar}(xt^{\sigma})\,dt$$

$$= \Gamma\beta \left(\frac{x}{2}\right)^{\aleph+2\hbar} {}_2\Psi_{m+2} \left[\begin{matrix} (\alpha + \sigma\aleph + 2\sigma\hbar, 2\sigma)(1,1) \\ (\hbar+1,1)\cdots(\hbar+1,1)(\aleph+\hbar+1,\psi)(\alpha+\beta+\sigma\aleph+2\sigma\hbar, 2\sigma) \end{matrix} \; ; \; \left(-\frac{x^2}{4}\right) \right] \tag{30}$$

Corollary 6.

Let $(z \in C/(-\infty,0], m \in N, \aleph, \hbar \in C \; and \; \psi > 0)$ Then by (14) the Laplace Transform, we obtain:

$$\int_0^\infty t^{\alpha-1} e^{-\delta t} J^{\psi,m}_{\aleph\hbar}(xt^{\sigma})\,dt$$

$$= \frac{1}{\delta^{\alpha}} \left(\frac{x}{2\delta^{\sigma}}\right)^{\aleph+2\hbar} {}_2\Psi_{m+1} \left[\begin{matrix} (\alpha + \sigma\aleph + 2\sigma\hbar, 2\sigma)(1,1) \\ (\hbar+1,1)\cdots(\hbar+1,1)(\aleph+\hbar+1,\psi) \end{matrix} \; ; \; \left(-\frac{x^2}{4\delta^{2\sigma}}\right) \right] \tag{31}$$

Corollary 7.

Let $(z \in C/(-\infty,0], m \in N, \aleph, \hbar \in C \text{ and } \psi > 0)$ Then by (18) the K-Transform, we obtain:

$$\int_0^\infty t^{\alpha-1} k_n(at) J_{\aleph,\hbar}^{\psi,m}(xt^\sigma)\,dt$$

$$= \frac{1}{4}\left(\frac{2}{a}\right)^\alpha \left(\frac{x}{2}\left(\frac{2}{a}\right)^\sigma\right)^{\aleph+2\hbar} {}_2\Psi_{m+1}\left[\begin{matrix}(\alpha+\sigma\aleph+2\sigma\hbar, 2\sigma)(1,1) \\ (\hbar+1,1)\cdots(\hbar+1,1)(\aleph+\hbar+1,\psi)\end{matrix} ; \left(-\frac{x^2}{4(\delta)^{2\sigma}}\right)\right] \quad (32)$$

Corollary 8.

Let $(z \in C/(-\infty,0], m \in N, \aleph, \hbar \in C \text{ and } \psi > 0)$ Then by (22) the Hankel Transform, we obtain:

$$\int_0^\infty t^{\alpha-1} J_n(at) J_{\aleph,\hbar}^{\psi,m}(xt^\sigma)\,dt$$

$$= \frac{1}{2}\left(\frac{x}{2}\right)^{\aleph+2\hbar}\left(\frac{2}{ak}\right)^{\alpha+\sigma(\aleph+2\hbar)} {}_2\Psi_{m+2}\left[\begin{matrix}\left(\frac{\alpha}{2}+\frac{\sigma\aleph}{2}+\frac{\eta}{2}+\sigma\hbar, \sigma\right)(1,1) \\ (\hbar+1,1)\cdots(\hbar+1,1)(\aleph+\hbar+1,\psi)\left(1+\frac{\eta}{2}-\frac{\alpha}{2}-\frac{\sigma\aleph}{2}-\sigma\hbar, -\sigma\right)\end{matrix} ; \left(-\frac{x^2}{4}\left(\frac{4}{a^2}\right)^\sigma\right)\right] \quad (33)$$

For m = k = ψ = 1 and $\hbar$ = 0, the corresponding corollaries are as given below

Corollary 9.

Let $(z \in C/(-\infty,0], \aleph \in C)$ Then by (10) the Euler Beta Transform, we obtain:

$$\int_0^\infty t^{\alpha-1}(1-t)^{\beta-1} J_\aleph^{1,1}(xt^\sigma)\,dt$$

$$= \Gamma\beta\left(\frac{x}{2}\right)^\aleph {}_1\Psi_2\left[\begin{matrix}(\alpha+\sigma\aleph, 2\sigma) \\ (\aleph+1,1)(\alpha+\beta+\sigma\aleph, 2\sigma)\end{matrix} ; \left(-\frac{x^2}{4}\right)\right] \quad (34)$$

Corollary 10.

Let $(z \in C/(-\infty,0], \aleph \in C)$ Then by (14) the Laplace Transform, we obtain:

$$\int_0^\infty t^{\alpha-1} e^{-\delta t} J_\aleph^{1,1}(xt^\sigma)\,dt = \frac{1}{\delta^\alpha}\left(\frac{x}{2\delta^\sigma}\right)^{\frac{\aleph+2\hbar}{k}} {}_1\Psi_1\left[\begin{matrix}(\alpha+\sigma\aleph, 2\sigma) \\ (\aleph+1,1)\end{matrix} ; \left(-\frac{x^2}{4\delta^{2\sigma}}\right)\right] \quad (35)$$

Corollary 11.

Let $(z \in C/(-\infty,0], \aleph \in C)$ Then by (18) the K-Transform, we obtain:

$$\int_0^\infty t^{\alpha-1} k_n(at) J_\aleph^{1,1}(xt^\sigma)\,dt = \frac{1}{4}\left(\frac{2}{a}\right)^\alpha\left(\frac{x}{2}\left(\frac{2}{a}\right)^\sigma\right)^\aleph {}_1\Psi_1\left[\begin{matrix}(\alpha+\sigma\aleph, 2\sigma) \\ (\aleph+1,1)\end{matrix} ; \left(-\frac{x^2}{4\delta^{2\sigma}}\right)\right] \quad (36)$$

Corollary 12.

Let $(z \in C/(-\infty,0], \aleph \in C)$ Then by (22) the Hankel Transform, we obtain:

$$\int_0^\infty t^{\alpha-1} J_n(at) J_\aleph^{1,1}(xt^\sigma)\,dt$$

$$= \frac{1}{2}\left(\frac{x}{2}\right)^{N}\left(\frac{2}{a}\right)^{\alpha+\sigma N} {}_1\Psi_2\left[\begin{matrix}\left(\frac{\alpha}{2}+\frac{\sigma N}{2}+\frac{\eta}{2},\sigma\right) \\ (N+1,1)\left(1+\frac{\eta}{2}-\frac{\alpha}{2}-\frac{\sigma N}{2},-\sigma\right)\end{matrix} ; \left(-\frac{x^2}{4}\left(\frac{4}{a^2}\right)^{\sigma}\right)\right] \quad (37)$$

CONCLUSION: The Euler Beta Transform,Laplace Transform,K-Transform,Hankel Transform for Generalized Lommel wright k-function were derived. When we take k = 1 we also deduce the results for Lommel wright function.

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